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DEVELOPMENT OF MATHEMATICAL MODELS FOR TEMPERATURE CONTROL OBJECTS IN THERMAL DESTRUCTION SYSTEMS BASED ON TRANSIENT PROCESS IDENTIFICATION

Maksym Maksymov
Department of Software and Computer-Integration Technology [1]

Oleksiy Kozlov✉
Department of Intelligent Information Systems [2]
kozlov_ov@ukr.net

Andriy Shynder
Department of Software and Computer-Integration Technology [1]

Oksana Maksymova
Scientific and Research Center "State Oceanarium" [3]

Anna Aleksieieva
Department of Ecology [2]

[1]Odesa Polytechnic National University
1 Shevchenko ave., Odesa, Ukraine, 65044

[2]Petro Mohyla Black Sea National University
10 68th Desantnykiv str., Mykolaiv, Ukraine, 54000

[3]Naval Institute of the National University "Odesa Maritime Academy"
8 Didrichson str., Odesa, Ukraine, 65029

✉**Corresponding author**

Abstract

Thermal destruction systems are essential for processing hydrocarbon waste, requiring precise temperature control to ensure operational efficiency and high product quality. However, accurately modeling the thermal behavior of such systems remains a challenging task due to the need for precise identification of transient heating characteristics. This study addresses this problem by developing an advanced two-point identification approach for constructing transfer function models of temperature control objects. Unlike conventional methods that rely on fixed reference points for each model order, the proposed approach leverages predetermined empirical coefficients, enabling flexible and more accurate identification from any two points on the transient characteristic, even in the presence of noise.

The effectiveness of the developed method is validated through the identification of two distinct experimental transient characteristics of a 100-liter reactor within a pilot thermal destruction system under noise conditions. Comparative analysis against three known identification techniques demonstrates that the proposed approach consistently yields the most accurate models while maintaining relatively low computational complexity, due to its versatility and ability to operate in noisy conditions. In particular, the values of the integral squared deviation were reduced by $0.126 \cdot 10^5$ for the first case and by $0.092 \cdot 10^5$ for the second case when compared with the best results achieved using the conventional two-point method. This confirms its high efficiency and the feasibility of using it to create comprehensive mathematical models of various systems for thermal destruction of hydrocarbon waste. The proposed approach is particularly well suited for rapid identification of transfer functions under varying operating conditions and can be applied to enhance control strategies, ultimately improving process stability, energy efficiency, and product quality in thermal destruction applications.

Keywords: thermal destruction system, temperature control, mathematical model, transfer function, transient process identification, two-point identification.

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1. Introduction

The development of advanced thermal destruction technologies plays a crucial role in hydrocarbon waste processing, enabling the conversion of complex organic compounds into valuable gaseous and liquid products [1–3]. Effective temperature regulation is a fundamental requirement for ensuring the stability and efficiency of such processes [4–6]. Thermal destruction reactors operate under highly dynamic conditions, where temperature variations significantly impact the reaction kinetics and product composition. Therefore, the creation of accurate mathematical models for automatic temperature control is essential for optimizing reactor performance, improving energy efficiency, and ensuring consistent product quality. Despite substantial advancements in control strategies, the accurate modeling of heating processes remains a challenge due to the nonlinear, non-stationary, and multivariable nature of the system dynamics [7, 8]. Addressing these complexities necessitates the development of refined mathematical models that capture transient process characteristics with high precision.

Traditionally, mathematical models for temperature control in thermal destruction systems have been constructed based on fundamental equations derived from the physics of heat transfer and chemical kinetics [9–11]. These approaches utilize energy balance equations, reaction rate laws, and thermodynamic principles to describe the dynamic behavior of the reactor [12, 13]. For instance, models based on Fourier's law of heat conduction and convective heat transfer coefficients have been widely applied to predict temperature distributions within reactors [14]. Additionally, chemical kinetics models incorporating Arrhenius-type expressions have been employed to simulate the rates of hydrocarbon decomposition reactions [15]. Although the considered models based on the fundamental equations of heat transfer and chemical kinetics possess a strong theoretical foundation, they are characterized by several critical limitations. Chief among these are the high computational demands and the inherent challenges in accurately determining key physical parameters of the temperature control object, such as heat transfer coefficients, reaction enthalpies, and specific heat capacities. Furthermore, these first-principles models often rely on a series of simplifying assumptions, such as ideal mixing, uniform temperature distribution, or steady-state operation [16–18], which can substantially diminish their ability to accurately capture the dynamic thermal behavior of real-world systems. These limitations significantly restrict the practical applicability of such models in the development of high-precision automatic temperature control systems for pyrolysis reactors in thermal destruction complexes and installations.

To overcome these limitations, an alternative methodology involves developing models based on the identification of experimentally obtained transient process characteristics, which is widely applied to various thermal objects [19–21]. This approach focuses on analyzing the system's response to various inputs to derive empirical models that capture the essential dynamics of the process. The most widely used models are those constructed using the identification of transient, impulse and frequency characteristics [22–24]. These models allow to derive system dynamics directly from observed process behavior, are less computationally intensive and can adapt to real-time data. Additionally, various artificial intelligence methods can be effectively used to approximate the dependencies of identified model parameters on changing input conditions and operating modes [25].

Despite the progress made in this direction, several challenges persist in achieving high-accuracy identification of transient responses, primarily due to measurement noise, sensor inaccuracies, and unmodeled disturbances [26]. Additionally, identification procedures must be both reliable and computationally simple, as the development of universal models for temperature control objects in thermal destruction systems of varying configurations often necessitates analyzing dozens or even hundreds of transient characteristics obtained under diverse conditions and subsequently generalizing the findings [27, 28]. These complexities highlight the need for further refinement of identification methodologies to enhance both accuracy and robustness.

For instance, in [25], a dynamic mathematical model of a pyrolysis reactor as a temperature control object is presented, developed through the identification of transfer function parameters based on transient heating characteristics. These identified dependencies were subsequently approximated using fuzzy logic. In this paper, a widely known approach was employed for the identification of the transient characteristics, utilizing a transfer function composed of two distinct

dynamic links with different time coefficients. A major limitation of this approach lies in its pronounced sensitivity to noise within the transient response, which severely hinders the accurate determination of the zero-crossing point of the second derivative curve. As a result, the inflection point must often be located graphically by constructing a tangent, a process that inherently introduces significant inaccuracies. These inaccuracies can propagate through the identification procedure, leading to substantial errors in the estimated model parameters. Additionally, the method is computationally intensive, as it involves repeatedly solving systems of transcendental equations for each new transient characteristic analyzed. This high computational burden restricts the method's practical applicability, particularly in scenarios requiring the processing of numerous experimental responses to construct a comprehensive and reliable model.

Another widely used identification method based on the Strejc model [29] employs a single inertial link of the *n*-th order, which significantly reduces computational complexity without compromising much accuracy. In this approach, a transfer function composed of multiple distinct aperiodic elements can be approximated by a single *n*-th order inertial link, requiring the identification of only one time constant instead of several. This constant, along with the model order, is selected from a reference table, eliminating the need to solve transcendental equations. However, despite its simpler implementation, the Strejc method also remains sensitive to noise, which complicates accurate detection of the inflection point on the experimental curve and introduces uncertainty in parameter estimation. Moreover, it does not independently account for the time delay constant, which may significantly degrade the accuracy of identification, especially when the object exhibits a pronounced transport delay. In such cases, errors in locating the inflection point can lead to incorrect determination of both the model order and its time constant.

The improved two-point identification method proposed in [30] can partially eliminate the shortcomings of the two previous approaches by eliminating the need for precise inflection point detection or tangent construction. Instead, it enables model identification using two characteristic points: the first corresponds to the theoretical inflection point of a model with a predefined order, and the second represents the moment the system reaches 90% or 80% of the steady-state temperature. This method somewhat enhances robustness to noise and allows for the determination of the model's order, main time constant, and delay time, as it utilizes a Strejc model with an added transport delay element. Despite offering improvements compared to the previous two methods, this approach still faces challenges in reliably identifying the required points under significant signal disturbances. Moreover, its reliance on fixed reference points, namely the model-specific inflection and a fixed percentage of the steady-state value, limits its flexibility. In some cases, the inability to clearly identify these specific points may necessitate alternative validation strategies.

Thus, the challenge of developing a universal approach for constructing comprehensive and adequate mathematical models of temperature control objects in thermal destruction systems based on reliable identification of transient characteristics in the presence of measurement noise, remains unresolved. In response to this gap, the primary objective of the present study is to develop and evaluate an extended and versatile approach for synthesizing high-fidelity mathematical models of such objects. This approach is intended to provide precise and efficient identification of transient heating characteristics, even under noisy conditions, using any two points on the experimental curve while also ensuring a more precise determination of these points. By increasing the accuracy of these models, the proposed approach will facilitate the design of more effective control strategies, ultimately improving process efficiency and product quality in the thermal destruction of hydrocarbon waste.

2. Materials and methods

The development of comprehensive models for temperature control objects within thermal destruction systems can be systematically structured into five generalized stages.

At the initial stage of mathematical model development, a specific temperature control object within the thermal destruction system is selected. These objects may include reactors – such as cyclic, continuous, or mixed-type configurations – where the thermal decomposition of waste occurs, as well as heat exchangers integrated into multi-loop recirculation systems and output

condensers responsible for the final separation of product fractions. Once the target object is identified, the primary input variable is defined, along with additional influencing factors that modify the model's parameters depending on operational conditions and modes. For instance, in a cyclic reactor, the primary input variable is the heating power supplied by the gas burner, while the waste loading level serves as a critical factor impacting model parameters [25]. Moreover, a variable characterizing the current composition of the waste used and the output heating temperature, which also changes the parameters of the model, can be additionally taken into account.

In the second stage, an appropriate model type with a generalized structure is selected, which will subsequently be synthesized through an identification method based on experimentally obtained transient process characteristics [26]. This identification process involves determining both the structural form and the specific parameters of the model. For instance, if a complex transfer function is chosen as the modeling framework, it is necessary to define the order of its components along with their respective parameters, such as gain and time coefficients [25].

The next stage involves determining all possible operational modes of the selected system. Experimental studies are then conducted under these conditions and modes on a real installation, during which transient process characteristics are recorded. These recorded data serve as the basis for refining and accurately identifying the parameters of the mathematical model [28].

The fourth stage involves the direct identification of the structure and parameters of the selected model for each experimentally obtained transient characteristic, corresponding to the analyzed operating modes and conditions. The accuracy of characteristic reproduction, the adequacy of the resulting model, and the computational complexity of the process are largely determined by the chosen identification method. An effective identification approach should ensure a balance between model precision and computational efficiency, enabling reliable representation of system dynamics across varying operational scenarios [27, 28].

At the final fifth stage, the individually identified models, each with precisely determined parameters and structures, are integrated into a comprehensive model capable of simulating the heating and cooling dynamics of temperature control objects within the thermal destruction system across all primary operating modes. This generalization is best achieved using fuzzy logic, artificial neural networks, or other machine learning techniques, depending on the number of available experimental characteristics and other features. These advanced methodologies enable accurate approximation of complex dependencies, ensuring the adaptability and reliability of the unified model [31–33].

This study primarily focuses on enhancing the fourth stage – identification of transient heating characteristics – by proposing an improved approach that ensures higher accuracy and efficiency in model formulation.

An approach using two links with different time coefficients. First, it is possible to examine a well-established approach for identifying the transient characteristics of thermal power systems, which offers moderate accuracy in model representation. According to this methodology, the temperature control object within the thermal destruction system is described by the transfer function $W_{\mathrm{TCO}}(s)$ including two different inertial links with different time coefficients [25]

$$W_{\mathrm{TCO}}(s)=\frac{T_{\mathrm{R}}(s)}{P_{\mathrm{H}}(s)}=\frac{Ke^{-\tau s}}{(T_1 s+1)(T_2 s+1)^n}, \tag{1}$$

where T_{R} is the output temperature of the control object; P_{H} is supplied input heating power; K is the gain coefficient; T_1 and T_2 denote the time coefficients of the aperiodic and inertial links, respectively; n corresponds to the order of the inertial link; τ represents the time delay.

To determine the parameters of this transfer function, a normalized experimentally obtained transient heating curve $T(t)$ of the object under specific initial conditions is utilized (**Fig. 1**). Additionally, based on this experimental characteristic, a graph of its second derivative $T''(t)$ is constructed to identify the inflection point ($t = t_i$), where the second derivative equals zero. In addition, **Fig. 1** also uses the following notations: $T(t_i)$ is the temperature value at the moment of inflection; T_{S} is the set temperature value after completion of the transient process; T_0 is the basic time constant. Further, the key parameters t_i, $T(t_i)$, T_0, and T_{S} are extracted from the transient characteristic $T(t)$.

Using these values, the system of equations (2) is solved to determine the time constant T_1 and the intermediate variable y

$$\begin{cases} \dfrac{T_1}{T_0} = e^{-y}; \\ y = \dfrac{t_i}{T_1}. \end{cases} \tag{2}$$

Subsequently, the system of equations (3) is solved to obtain the intermediate variable x, allowing the time constant T_2 to be calculated as $T_2 = T_1/x$

$$\begin{cases} xe^{-y} = e^{-y/x}; \\ (1+x)e^{-y} = 1 - \dfrac{T(t_i)}{T_S}. \end{cases} \tag{3}$$

Additionally, the gain factor K is determined as the ratio of the steady-state output temperature T_S to the input heating power P_H after the transient process has completed. The obtained values of K, T_1, and T_2 are then substituted into the transfer function, and an initial simulation is conducted with a time delay $\tau = 0$ and an inertial link's order $n = 1$. The resulting response is compared to the experimental characteristic, and if their inflection points (t_i and t_{im}) do not match, a time delay τ is introduced, calculated as $t_i - t_{im}$. To further improve accuracy, the model order n is systematically varied (e.g., $n = 2, 3, 4$, etc.), with each iteration simulated and evaluated using a predefined proximity criterion, such as the mean square deviation or other. The optimal order n is then selected based on the best fit to the experimental data.

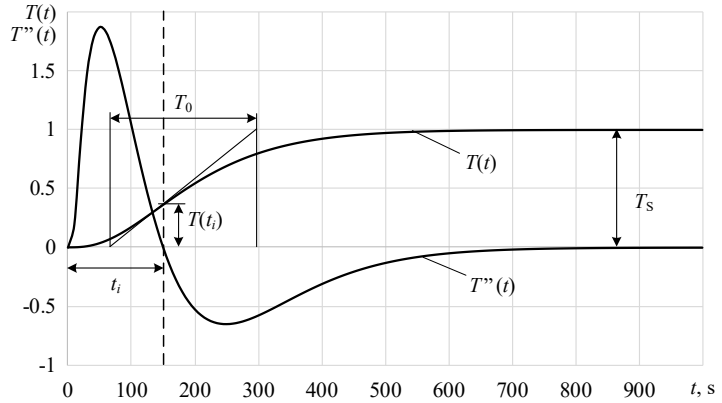

Fig. 1. Transient response characteristics of a temperature control object

The primary drawback of this approach is its susceptibility to noise in the transient response, which makes it nearly impossible to precisely determine the intersection of the second derivative graph with the time axis. Consequently, the inflection point must be sought graphically by constructing a tangent, introducing a considerable degree of inaccuracy. As a result, all identified model parameters may exhibit significant errors. Furthermore, this method entails substantial computational effort, as it requires solving systems of transcendental equations each time a new transient response is analyzed. This imposes considerable limitations on its applicability, particularly when it is necessary to process a large number of experimental characteristics to develop a comprehensive model.

An approach using the Strejc model. The Strejc method, which uses only one inertial link of the n-th order, has significantly lower computational costs and no worse accuracy. According to this method, a transfer function composed of n distinct aperiodic links, each with different time constants, can be enough accurately approximated by a single inertial link of the n-th order [29]

$$W_{TCO}(s) = \frac{T_R(s)}{P_H(s)} = \frac{K}{(T_1 s+1)(T_2 s+1)...(T_n s+1)} \cong \frac{K}{(Ts+1)^n}. \tag{4}$$

Thus, during the identification process, only a single time constant needs to be determined instead of two. Moreover, rather than solving systems of transcendental equations, this constant T, along with the model order n, is obtained from a predefined reference table (**Table 1**).

Table 1
The reference table for determining model's parameters (the Strejc method)

n	1	2	3	4	5	6	7	8	9	10
t_a/t_b	0	0.104	0.218	0.319	0.41	0.493	0.57	0.642	0.709	0.773
t_e/t_b	1	0.736	0.677	0.647	0.629	0.616	0.606	0.599	0.593	0.587
$T(t_i)$	0	0.264	0.323	0.353	0.371	0.384	0.394	0.401	0.407	0.413
t_a/T	0	0.282	0.805	1.425	2.1	2.811	3.549	4.307	5.081	5.869
t_b/T	1	2.718	3.695	4.463	5.119	5.699	6.226	6.711	7.164	5.59
t_e/T	1	2	2.5	2.888	3.219	3.51	3.775	4.018	4.245	4.458
t_d/T	0	1	2	3	4	5	6	7	9	9

Initially, the parameters t_a, t_b, t_e, and t_d are extracted from the experimentally obtained transient response graph (**Fig. 2**).

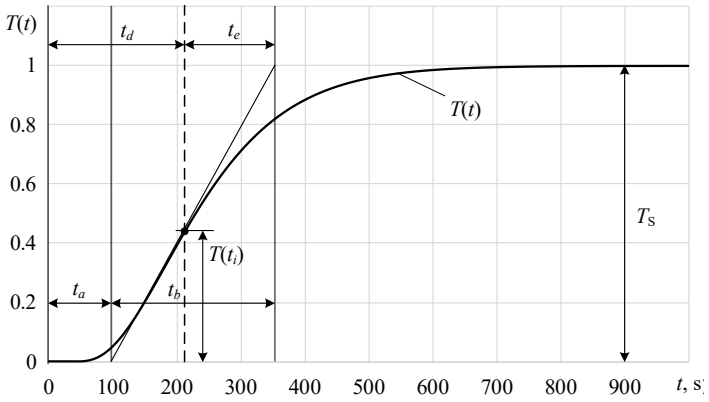

Fig. 2. Determination of parameters t_a, t_b, t_e, and t_d from transient response

The model order n is then determined based on the ratios t_a/t_b and t_e/t_b, followed by the calculation of the model's time constant T using the relations t_a/T, t_b/T, t_e/T, and t_d/T for this determined order. Moreover, the order of the model n and its time constant T can be determined only by the first relations t_a/t_b and t_a/T, and the remaining relations are used only for additional verification. In case of obtaining intermediate values of the relation t_a/t_b, which are not in the table, the nearest smaller number is selected. The gain factor K is determined in the same way as in the previous approach.

Despite its low computational cost, this method, like the previous approach, encounters challenges in accurately identifying the inflection point and subsequent parameters on the experimental characteristic in the presence of noise. Furthermore, it does not account for the separate determination of the model's time delay constant τ, which may also lead to a significant reduction in identification accuracy. In cases where the object exhibits a distinct and significant transport delay, the inflection point may be determined incorrectly with subsequent erroneous determination of both the model's order and its time constant.

Two-point identification method. To overcome the primary limitations of the two previously discussed approaches, an improved method is proposed in [30]. This method eliminates the need for precise inflection point determination or tangent construction and instead enables identification based on two points of the experimental characteristic. The first point corresponds to the assumed inflection, not on the experimental characteristic itself, but on the model characteristic with a predefined order. The second point is defined as the moment when the system reaches either 90% or 80% of the steady-state temperature value.

This method allows for fairly effective identification in the presence of noise and involves determining the delay time constant τ in addition to the main time constant T and the order of the model n, since its approximating transfer function is the Strejc model together with the transport delay link [30]

$$W_{\mathrm{TCO}}(s)=\frac{T_{\mathrm{R}}(s)}{P_{\mathrm{H}}(s)}=\frac{Ke^{-\tau s}}{(Ts+1)^{n}}. \tag{5}$$

In study [30], the formulas were derived to implement this method, enabling the determination of both the basic time constant T and the delay time constant τ for models up to the fifth order (**Table 2**). These formulas were obtained based on known equations describing the model's response to a step input, along with its first and second derivatives.

Given that the object's order is initially unknown, the model parameters T and τ are identified for each possible order (ranging from $n=2$ to $n=5$) using the formulas provided in **Table 2**.

Table 2
The reference table for determining model's parameters (the Two-point method)

n	$T(t_i)$	T, using the point 90% of the steady-state value t_{90}	T, using the point 80% of the steady-state value t_{80}	τ
2	0.264	$T=0.34605\,(t_{90}-t_i)$	$T=0.50143\,(t_{80}-t_i)$	$\tau=t_i-T$
3	0.323	$T=0.30099\,(t_{90}-t_i)$	$T=0.43878\,(t_{80}-t_i)$	$\tau=t_i-2T$
4	0.353	$T=0.27168\,(t_{90}-t_i)$	$T=0.39761\,(t_{80}-t_i)$	$\tau=t_i-3T$
5	0.371	$T=0.25040\,(t_{90}-t_i)$	$T=0.36751\,(t_{80}-t_i)$	$\tau=t_i-4T$

Subsequently, the integral squared deviation between the model's transient response and the experimental characteristic is computed for all considered orders. The optimal model is then selected with the order n that yields the smallest integral squared deviation, ensuring the best fit to the experimental data.

Although this method offers higher identification accuracy compared to the two previously discussed approaches, it remains susceptible to significant errors in determining the required points on the graph under strong noise influence. Another key limitation is that identification is restricted to predefined points – namely, the inflection point (t_i) for each order and the 90% (t_{90}) or 80% (t_{80}) of steady-state value. In certain cases, locating these specific points may be challenging, necessitating the use of alternative points for validation. To overcome these limitations, this study proposes an extended and more versatile approach to creating models for the temperature control objects of thermal destruction systems. The developed approach enables identification using any two points on the experimental curve while also ensuring a more precise determination of these points.

3. Results and discussion

Extended two-point identification approach. The proposed approach relies on the determination of empirical coefficients $K^{n}_{t_1/t_2}$, which are computed as the ratio of two time moments, t_1 and t_2, at which the experimental transient response reaches specific percentage values T_1/T_{S} and T_2/T_{S} of the steady-state temperature. These coefficient sets can be precomputed for all relevant time moments pairs and model orders n through preliminary computational experiments with generalized transfer functions of various orders of the form (5). During these simulations, the corresponding time moments can be automatically recorded when the transient response reaches the predefined percentage values, using specialized software. Consequently, during the identification process, the model order is determined based on the value of coefficient $K^{n}_{t_1/t_2}$. Subsequently, for the accurate determination of the time constant T in the transfer function (5) for the identified order, an additional coefficient K^{n}_{T/t_2} is introduced, which represents the ratio of the time constant T to the time moment t_2. Similar to the first set, the values of these coefficients for different orders and predefined time moments must also be precomputed.

Furthermore, when applying this approach to real transient characteristics affected by noise, multiple crossing points of the first and second percentage values may be recorded, resulting in determining multiple variants of time moments t_1 and t_2, as illustrated in **Fig. 3**. In such cases, to determine the final time values, it is recommended to average these points using established statistical methods, for example, computing the arithmetic mean

$$t_1 = \frac{\sum_{j=1}^{m} t_{1j}}{m}, \tag{6}$$

where m is the total number of recorded moments of intersection of the percentage value T_1/T_S.

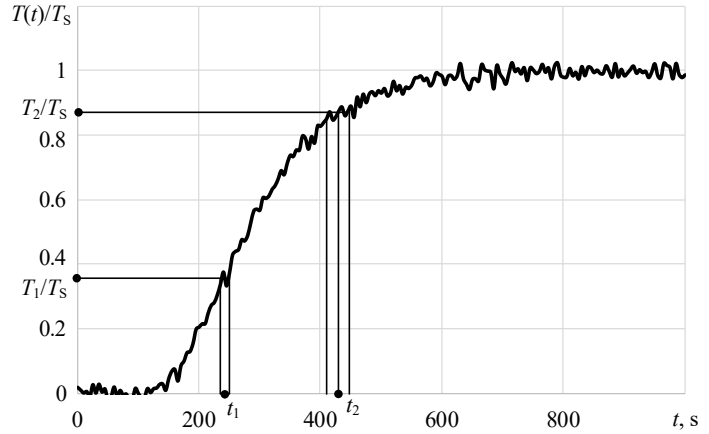

Fig. 3. Determination of time moments t_1 and t_2 under noise conditions

In turn, for the time moment t_2, the averaging is carried out in a similar manner. For instance, for two selected pairs of time moments (t_{30} and t_{90}; t_{25} and t_{75}), the corresponding sets of coefficients $K^n_{t_1/t_2}$ and K^n_{T/t_2} are determined for model orders ranging from 1 to 11 and presented in **Table 3**.

Table 3
The reference table for determining model's parameters (extended two-point method)

n	$K^n_{t_{30}/t_{90}}$	$K^n_{T/t_{90}}$	$K^n_{t_{25}/t_{75}}$	$K^n_{T/t_{75}}$
1	0.155	0.434	0.208	0.72
2	0.282	0.257	0.357	0.371
3	0.359	0.188	0.441	0.255
4	0.413	0.149	0.496	0.196
5	0.455	0.125	0.537	0.159
6	0.487	0.108	0.569	0.135
7	0.514	0.095	0.594	0.117
8	0.536	0.084	0.615	0.103
9	0.556	0.077	0.633	0.093
10	0.572	0.07	0.648	0.084
11	0.587	0.065	0.662	0.077

Given that the presence of a pure transport delay in the object is initially unknown, it must be considered when computing the empirical coefficients based on the experimental characteristic. For instance, in the case of the coefficient $K^n_{t_1/t_2}$

$$K^n_{t_1/t_2} = \frac{t_1 - \tau^n}{t_2 - \tau^n}. \tag{7}$$

Following straightforward transformations, the delay time τ^n for the model of the n-th order can be determined as follows

$$\tau^n = \frac{t_1 - K^n_{t_1/t_2} t_2}{1 - K^n_{t_1/t_2}}. \quad (8)$$

Thus, the proposed approach to identifying the experimental transient response of a given temperature object is implemented through the following sequence of steps. First, transfer functions are generated for each of the considered model orders. If an initial hypothesis regarding the potential order of the object is available, the number of models can be significantly reduced. Next, the time points t_1 and t_2 are taken from the experimental characteristic (if multiple values are recorded for each point, the data are averaged using formula (6)). Subsequently, for all considered models of the selected orders, the delay time constants τ^n are computed using formula (8), while the basic time constants T^n are determined according to formula (9)

$$T^n = K^n_{T/t_2} \left(t_2 - \tau^n \right). \quad (9)$$

The gain factor K of all the obtained transfer functions is determined in the same way as for all the methods considered in the previous section.

Subsequently, all synthesized models undergo simulation, and their transient response characteristics are evaluated against the experimental characteristic of the object using the integral square deviation criterion. The optimal model, corresponding to the correct order, is identified based on the minimal deviation value. If additional validation of the obtained results is required, the entire identification process can be reiterated using an alternative pair of time points t_1 and t_2.

Validation of the proposed approach. To validate the effectiveness of the proposed approach, this study applies it to the identification of two distinct experimental transient characteristics (**Fig. 4, *a***, ***b***) recorded during the heating process of a 100-liter reactor in an experimental thermal destruction system. The first characteristic corresponds to a supplied heating power of 17 kW with an initial reactor loading of 90%, while the second was obtained with a heating power of 22 kW and an initial loading of 50%. For comparative analysis, identification was performed for both cases using the proposed approach, alongside the three methods discussed in the previous section.

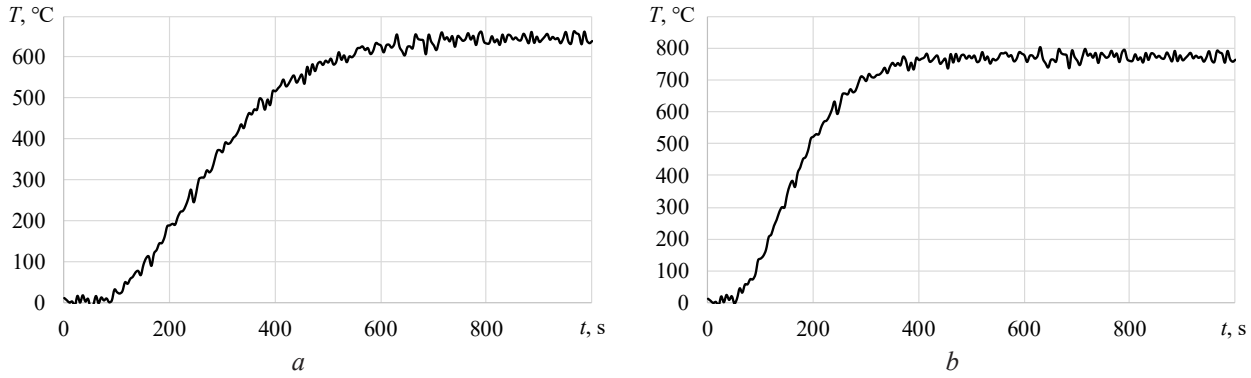

Fig. 4. Experimental transient characteristics for the reactor heating of the pilot thermal destruction system: *a* – heating power of 17 kW and an initial reactor loading of 90%; *b* – heating power of 22 kW and an initial loading of 50%

Using the two-point identification method, calculations were performed for model orders n ranging from the 2nd to the 5th, followed by the selection of the most accurate model. In this case, the second reference point was the time moment t_{90}. In turn, the proposed extended two-point approach considered model orders n from the 1st to the 10th, also with further selecting the optimal model. For the identification of transient characteristics, the time moments t_{30} and t_{90} were utilized, with formula (6) applied to average their different values.

Table 4 summarizes the results of the experimental transient characteristics' identification process, including the defined transfer functions of the models with their respective parameter values, as well as the integral square deviation values used for adequacy analysis.

Additionally, **Fig. 5** presents the simulation results of the identified transfer functions obtained using all the studied identification methods, juxtaposed with the experimental characteristics for the first case (heating power of 17 kW and initial reactor loading of 90%).

Table 4

Results of the experimental transient characteristics' identification process for the 100-liter reactor of the pilot thermal destruction system

Model properties	Identification methods under study			
	Method using two different links	**Strejc method**	**Two-point method**	**Extended two-point method**
1st characteristic: heating power of 17 kW, initial reactor loading of 90%				
Obtained transfer function	$\frac{0.038e^{-125s}}{(67.6s+1)(23.9s+1)^3}$	$\frac{0.038}{(59.6s+1)^5}$	$\frac{0.038e^{-4.5s}}{(74.3s+1)^4}$	$\frac{0.038e^{-8.77s}}{(72.6s+1)^4}$
Integral square deviation value	$2.786 \cdot 10^6$	$1.77 \cdot 10^5$	$1.346 \cdot 10^5$	$1.22 \cdot 10^5$
2nd characteristic: heating power of 22 kW, initial reactor loading of 50%				
Model properties	**Identification methods under study**			
	Method using two different links	**Strejc method**	**Two-point method**	**Extended two-point method**
Obtained transfer function	$\frac{0.035e^{-54s}}{(66.3s+1)(16.51s+1)^3}$	$\frac{0.035}{(54.14s+1)^4}$	$\frac{0.035e^{-13.1s}}{(40.9s+1)^4}$	$\frac{0.035e^{-5.4s}}{(43.15s+1)^4}$
Integral square deviation value	$6.362 \cdot 10^5$	$3.841 \cdot 10^6$	$1.648 \cdot 10^5$	$1.556 \cdot 10^5$

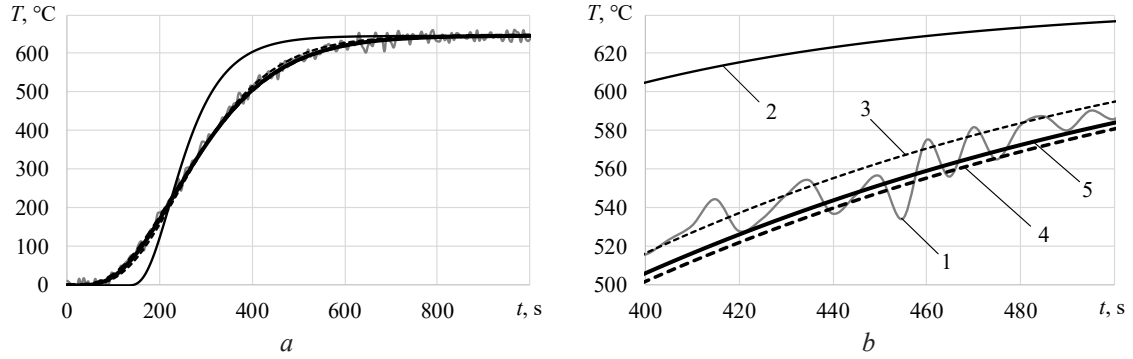

Fig. 5. Simulation results of the obtained models using the studied methods for the first case: *a* – general view; *b* – detailed view; 1 – real process; 2 – method using two different links; 3 – Strejc method; 4 – two-point method; 5 – extended two-point method

In the same way, **Fig. 6** shows the simulation results for the second case (heating power of 22 kW and initial loading of 50%).

As evidenced by **Table 4** as well as **Fig. 5, 6**, the models synthesized using the proposed extended two-point identification approach demonstrate the highest accuracy compared to those developed with the other studied methods in both considered cases. The two-point method introduced in paper [30] also yields reliable results, albeit with slightly lower accuracy. In contrast, models obtained using the method with two different links and the Strejc method exhibit significantly reduced precision – the former showing the poorest performance in the first case, while the latter underperforms in the second case.

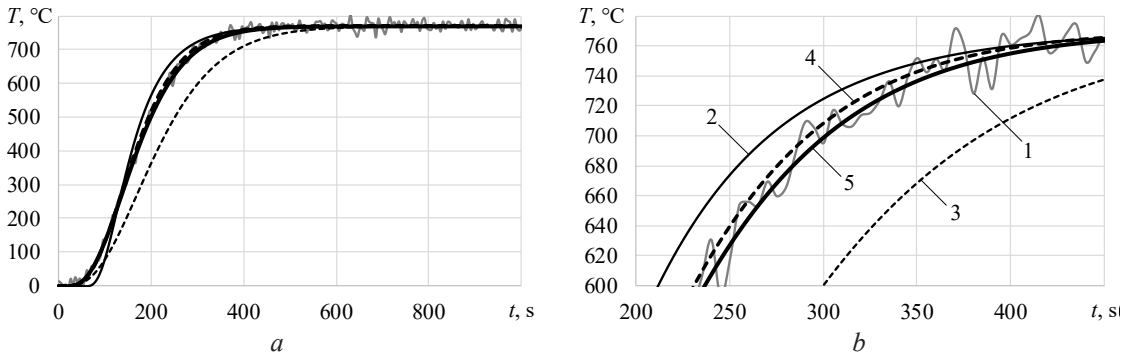

Fig. 6. Simulation results of the obtained models using the studied methods for the second case: *a* – general view; *b* – detailed view; 1 – real process; 2 – method using two different links; 3 – Strejc method; 4 – two-point method; 5 – extended two-point method

Despite the slightly lower accuracy of the conventional two-point method, the developed extended approach offers greater universality, as it enables identification using any two convenient points on the transient characteristic. Furthermore, this method maintains relatively low computational complexity, making it suitable for rapid identification of multiple transfer functions based on various experimental characteristics taken from a temperature control object in various operating modes.

These identified models can subsequently be integrated into a unified complex model, for instance, using fuzzy logic [33–35]. Additionally, different pairs of points on the experimental characteristic can be leveraged for further verification of the obtained model parameters. Overall, the findings confirm the high efficiency of the proposed extended approach for identifying transfer functions based on experimental characteristics of temperature control objects (including when working with noisy data) and its practical applicability in creating comprehensive mathematical and computational models for thermal destruction systems.

The proposed approach has certain limitations that should be acknowledged.

Experimental transient characteristics of temperature objects must be recorded during a dedicated active identification experiment, conducted without automatic control loops and under minor disturbances. If the system operates under automatic control or is subject to significant external disturbances, accurate identification using this method will not be feasible.

Additionally, the approach is constrained in its applicability to objects exhibiting significant non-stationarity or non-linearity. However, in such cases, an alternative strategy can be employed – by selecting different pairs of points in distinct sections of the transient response, it is possible to determine the dynamic variations of model parameters as they change with time or temperature. This adaptive refinement of the method is planned for further investigation in future studies.

4. Conclusions

This study introduces and analyzes an advanced approach for developing accurate and comprehensive mathematical models of temperature control objects within thermal destruction systems, based on the identification of transient heating characteristics. The proposed approach enables the construction of models in the form of transfer functions with precisely defined parameters and order, leveraging a two-point identification procedure applied to experimental transient characteristics of temperature control objects. In contrast to the known conventional two-point identification method, which rely on fixed reference points for each model order, the presented approach offers greater flexibility and universality. By utilizing predetermined empirical coefficients, it allows identification to be performed using any two points on the characteristic, including in the presence of noise, thereby enhancing its adaptability and robustness.

The effectiveness and validation of the proposed extended two-point identification approach are assessed through the identification of two distinct experimental transient characteristics of a 100-liter reactor within a pilot thermal destruction system. In turn, the first characteristic was

obtained at a supplied heating power of 17 kW with an initial reactor loading of 90%, the second one was recorded for a heating power of 22 kW and an initial loading of 50%. The evaluation is performed in comparison with three alternative methods: the conventional two-point identification method, the Strejc method, and an approach utilizing two different links. The simulation results demonstrate that models synthesized using the proposed method exhibit the highest accuracy across both considered cases, which corresponds to the smallest values of the integral square deviation: $1.22 \cdot 10^5$ for the first experiment and $1.556 \cdot 10^5$ for the second experiment. Thus, when compared with the best results achieved using the conventional two-point method, the values of the integral squared deviation were reduced by $0.126 \cdot 10^5$ (first case) and by $0.092 \cdot 10^5$ (second case). Additionally, the approach maintains relatively low computational complexity, enabling the rapid identification of multiple transfer functions based on various experimental characteristics obtained from a temperature control object operating under different conditions. These identified models can subsequently be integrated into a unified, comprehensive model that captures the full dynamic behavior of temperature control objects across all primary operating modes, utilizing techniques such as fuzzy logic, neural networks, or other machine learning algorithms.

Overall, the obtained results confirm the high accuracy and robustness of the proposed extended approach in identifying transfer functions from experimental transient characteristics of temperature control objects, even under noisy conditions. Its practical applicability extends to the development of various mathematical and computational models for thermal destruction systems, enabling a more precise representation of system dynamics. By improving the fidelity and flexibility of the identified models, this approach supports the design of more efficient control strategies, ultimately enhancing process stability, operational efficiency, and product quality in the thermal destruction of hydrocarbon waste.

Conflict of interest

The authors declare that they have no conflict of interest in relation to this research, whether financial, personal, authorship or otherwise, that could affect the research and its results presented in this paper.

Financing

The study was performed without financial support.

Data availability

Manuscript has no associated data.

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies when creating the current work.

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