Development of an Artificial Intelligence Tool and Sensing in Informatization Systems of Mobile Robots[1]⋆

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Abstract
The informatization systems of mobile robots are considered. A mock-up of an autonomous tool for research and modeling of the navigation system, movement models, informatization systems, manipulators and sensing of mobile robotic systems has been developed. The system is built on the JetRacer platform, which uses NVIDIA Jetson Nano. Peculiarities of representation of nonlinear differential models by a recurrent algebraic sequence are studied. The influence of the linearization error of the automated control system (ACS) object, the vector-indicator on the model error was studied and analytically evaluated. A method of model shape transformation has been developed, as a result of which we have three new formalized information quantities. Analytical expressions of the approximation of the solution and the upper limit of the error and the number of iterations, starting from which the error will be smaller than the given one. It is shown that its suitability for information analysis and conclusion, without human participation, defines it as an artificial intelligence (AI) tool. The influence of the linearization error of the ACS object, the indicator vector on the model error was studied. A kinematics model of the manipulator is formed based on the analytical solution of the inverse problem and the sensing system, which determines the angular position of the gripper. Simulation of robot movement, influence of the number of eigenvalues and model parameters on the error was carried out, and the error of the solution of the inverse problem was investigated.

Keywords
Nonlinear Model, Error Estimation, Recurrent Sequence, Informatization of Estimation, Artificial Intelligence Tool

1. Introduction

The successes demonstrated in the development of unmanned technologies, namely mobile robotics [1, 5] and the growing need for them in society [2, 3] stimulate the search for technical means of intellectual improvement to expand functional capabilities [2, 3]. One of the examples of solving a set of problems to improve the movement of vehicles and ensure efficiency and safety on the roads are analogues of autonomous route planning and navigation in transport systems: Waymo, Tesla Autopilot, Uber ATG, Baidu Apollo, Mobileye. Being the development of various campaigns, they are aimed at introducing autonomous driving technologies. One of the key components of the Waymo system is the LiDAR sensor, which creates accurate 3D maps of the environment. Waymo uses machine learning algorithms to recognize objects and other vehicles for non-emergency interaction with them. It takes complex data gathered from a leading set of sensors and analyzes the environment. Despite the possibility of borrowing technical solutions for the development of air, surface and underwater mobile robotic technical means, specific problems arise that

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complicate their application. They are caused by structural differences in the equations of motion, influences of added masses, the nature of resistance and infiltration. At the same time, the difference in models suitable for simple calculations leads to differences in the methods of solving dynamics problems and requires new searches for intellectualization and hardware and software improvement of sensory systems.

2. Analysis of literature data on the results of recent research

The practical implementation of mobile devices demonstrates the need for a comprehensive hardware and software solution to navigation problems. One example is Tesla Autopilot. It uses a combination of cameras, radars and sensors to navigate the vehicle [1]. The recognition of road signs, the control of the movement of cars within the lane and the execution of maneuvers use the principles of machine learning to adapt to different conditions of the road lane [1]. Any Tesla built since the end of 2021 is equipped with 8 cameras, 12 ultrasonic sensors, vision processing tools. The implementation of algorithms for the analysis of visual images of a set of cameras allowed the removal of ultrasonic sensors from Models 3, Y, and in 2022 from Model X and S. The experience of Uber ATG [2] confirms this conclusion for autonomous road transport and taxis. Today, unmanned vehicles use machine learning models that allow them to drive safely and precisely [1, 2]. It becomes especially difficult to control the movement of individual adaptation during the accompaniment of social security robots supporting the elderly [3]. Components are trained by one or more machine learning models. All components together form the software for unmanned vehicles: perception, prediction, planning and control of movement and suitable for extension to other types of robots. The "control" component, which controls the wheels of the unmanned vehicle and controls the brakes and accelerator, is special, as it is directly applicable only to ground operations [4]. The work [5] considered systems that must be applied comprehensively. It is shown that distance sensors with a narrow beam will gain an advantage [5]. It should be expected that the review of the frequency range of acoustic waves will open the possibility of using new sound systems in combination with video and laser systems [6]. Another view of the problems of mobile robot navigation is highlighted in [7]. To implement movement along a pre-planned route, in the space where there are no road boundaries, current re-planning is required, if there is a predictive assessment of disturbances based on the perception data of forecast information received from the sensors. The authors propose a cognitive model of perception as a solution to situational management problems using replanning of the movement route. However, its ability to meet the requirements for autonomous systems is limited by verbal forms of representation [7]. To eliminate the shortcomings of verbal fuzzy rules, the Takagi-Sugeno-Kang fuzzy inference model was applied [8]. Verification of operation in noisy conditions, derivation of level 2 rules, for the cartographic base of knowledge about the environment and the construction of traffic routes. These are far from all the advantages obtained on the basis of the Takagi-Sugeno-Kang fuzzy inference model [8].

However, the time for training and retraining is a limitation and a disadvantage, therefore calibration is one of the simple and reliable solutions [9]. In [10], simultaneous application of recurrent network technologies and analytical determination of weight coefficients is proposed. In addition, it is shown that the presence of an analytical model of the dynamics of elements, including obstacles, significantly simplifies the solution of the problem as a whole [10]. A hardware solution that simplifies the task due to the use of three processor components, programmable logic and a software processor is promising [11]. Its generality and mobility allows you to adjust the parameters of the source code and adapt it to work with various sensors [11]. This will be especially important if a recursive predicator is used to form neural network commands for controlling mobile robots [12]. It is obvious that it is possible to ensure the implementation of approaches [10 - 12] under the conditions of using models of spatial movement of mobile robots in the inertial coordinate system [13]. Its presentation in forms suitable for fast express calculations provides a significant advantage for such problems [13]. However, the direct application of these works is hindered by the need for automatic preparation of data for the analysis of properties about the object and processes for the formation of a set of logical rules and an AI tool for the formation of a conclusion.

Analysis of the state and forecast and prospects for the development of robotics, automated and information systems, presented in correlation with future forecasts and perspectives of the needs and transformation of the culture of sports and life science, is evidence of the search for new AI tools [14]. Further development of AI methods and algorithms as a promising tool for forming requirements for the design of sensors and robot control systems with improved technical characteristics is devoted to the work [15]. The presented machine learning algorithm is based on the idea of building a model from data samples for predictions and/or decision-making [15]. An overview of the process models of recent years allows us to state that a general revisionary attitude to the fundamental integral-differential calculus is manifested in most models. Uncertainty, as a property of modern types of models, is becoming more and more dominant [16]. The experience and success of the fuzzy system will serve as an alternative prototype for further development of a simplified representation of complex models [16]. Their further development will be accompanied by the need to implement a generalized cyber-physical system [17]. As the authors of the work [17] prove, early diagnostics will become a priority for industrial equipment based on Industry 4.0 standards. The Internet of Things as a necessary component will also play its role in the formation of new requirements for compression and protection of information [17].

An equally important task compared to considering the forms, advantages and disadvantages of models is the need to assess methodological and instrumental errors [18]. Due to the fact that these component errors are values of multifactorial influence, their assessment ensures the informational completeness of experimental data [18]. Attempts to estimate the tolerance as a quantitative interval for problems of estimating the parameters of radio electronic circuits by the method of ellipsoidal estimation are presented in [19]. However, it was not possible to realize the full advantages of using the method, which opens up the possibility of parallel consideration [19]. Their cause is quantization during the digital transformation of analog values. At the same time, an important need is the diagnosis of aging and dislocations of lattice electrons and structural changes that precede sensor failures, as a result of which changes in characteristics and noise generation occur [20]. A review of modern methods of early detection of such deviations together with the use of differential inclusion and processing schemes partially solves this problem [20]. However, its experimental solution requires duplication, which increases the cost of the system as a whole. In this regard, as shown in [21], the problem of structuring, periodic testing by a set of points becomes a necessary function of reliability control of hardware used to process information flows and assess reliability [21].

Thus, the main unsolved task is the development of a tool for automatic preparation of data about an object and processes to form a description of its properties and tools for a set of logical rules or an AI tool for forming a conclusion and correction of sensory data.

3. The purpose and objectives of the research

The aim of the study was to establish the properties and form the AI tools of analytical nonlinear models and to propose methods of their transformation into algebraic forms of successive approximations that coincide and are suitable for fast express calculations. This will provide an opportunity to build movement models of mobile robotics based on the laws of dynamics and implement the prescription paradigm in the presence of obstacles and limitations.

To achieve this goal, the following tasks were formulated:

- to form an example of a mobile robot for researching the means of intellectualization and modification of sensory control systems;
- form and investigate the representation of nonlinear differential models to a recurrent algebraic sequence by serial expansion with three-level comparators and vector-indicator;
- to form a kinematic model of the manipulator based on the analytical solution of the inverse kinematics problem for unity solution.

4. Development of an analytical model, an artificial intelligence tool for informatization of mobile robotic systems.

4.1 Formation of the research layout of means of intellectualization and sensitization of mobile robotic systems.

An autonomous navigation system built on the JetRacer platform using the NVIDIA Jetson Nano was taken as a tool for research and simulation. The main components of which were taken: IMX219 8 MP HD, wide -angle camera with a viewing angle of 160°; Oled display 0.91" 128×32 pixels. AC8265 wireless network, dual-band Wi-Fi 2.4G/5G, Bluetooth 4.2. High speed 4WD engines. Front and rear axle differentials. Battery 8.4V, 18650 × 4 (two in parallel, two in series). RC380 high speed carbon brush motor Idle speed 15000 rpm. Servo drive. The torque is 6 kgcm. The kit includes an expansion board with its own battery, charge control device and its activated protection. The expansion board controls the wheel servo, it has a motor, Jetson Nano, cooling fan, wireless network antennas, dual-band Wi-Fi 2.4G/5G. The 128-core Jetson Nano GPU, using Nvidia's Maxwell architecture, is capable of delivering 472 GFLOPS combined with 4GB of RAM and a quad-core ARM A57 processor, and 472 billion of them define the computer's capabilities. An SD card with a capacity of at least 64G and a JetRacer image downloaded using the Etcher image burning program. The Jetson Nano has a 40-pin GPIO. If necessary, functioning LEDs and switches, input and output ports are connected to the GPIO ports. For physical modeling, it was planned to connect three MPU-6050 GY-521 sets - a 3-axis accelerometer and a 3-axis gyroscope controlled by the I2C (TWI) protocol. Its main purpose was to determine the orientation of the grip of the manipulator in the space.

4.2 Representation of nonlinear differential models by a recurrent algebraic sequence

A. Formulation and solution of the problem of estimating the influence of the linearization error of object models on the model error. The generalized problem of modeling a non-linear object was considered. A more common nonlinear model presented in differential form was also chosen. Assume that it can be divided into two parts by introducing linear $L_1(\Phi)$ and nonlinear $L_2(\Phi)$ differential operators in the form:

$$L_1(\Phi)=\sum_{k=0}^{l} a_{l-k}\frac{d^k\Phi}{dx_j^k}; \quad (1)$$

$$L_2(\Phi_1+\Phi_2)\neq L_2(\Phi_1)+L_2(\Phi_2); \quad (2)$$

$$L_1(\Phi)=L_2(\Phi). \quad (3)$$

Suppose that there is a finite-integral transformation of the image $L_1(\Phi)$ and which represents it as a linear function of the transformed Φ with the same kernel K_{nj} and eigenvalues, σ_{nj} i.e.:

$$\int_a^b K_{nj}L_1(\Phi)dx_j = \sum_{k=0}^{l} a_{l-k}\sigma_{nj}^k\int_a^b K_{nj}\Phi dx_j = \tilde{\Phi}\sum_{k=0}^{l} a_{l-k}\sigma_{nj}^k. \quad (4)$$

Such a model is able to describe most of the processes to be managed according to the principles of ACS TP in CS and the technologies they provide [10]. Acting by analogy to the solution of problem (3), taking into account the properties of continuity and differentiability of the generalized operator $L_2(\Phi)$, we determine by the same algorithm [13] the first approximation of the function Φ_1, as a solution:

$$L_1(\Phi_1)=L_3(\Phi_1).$$

Next, we denote the error of the right-hand side as a consequence of the approximation of the nonlinear operator δ:

$$\delta=L_2(\Phi_1)-L_3(\Phi_1) \quad (5)$$

Based on the linearity of the left part and taking into account the notation, we write:

$$L_1(\Delta)=\delta.$$

The right-hand side of the latter formally coincides with the expression given in the original problem. However, such a transformation made the problem more convenient for estimating the error arising as a result of linearization. Thus, the application of the finite-integral transformation to the problem of finding the error gives:

$$\tilde{\Delta}\sum_{k=0}^{l} a_{l-k}\sigma_{nj}^k=\int_a^b K_{nj}\delta dx_j. \quad (6)$$

The advantage of the steps taken is the suitability of (6) for estimating the upper bound of the right-hand side of (5), which can be done taking into account the Buniakovsky and Cauchy inequalities [22] and the condition of normalization of kernels, namely:

$$\left|\int_a^b K_{nj}\delta dx_j\right|\leq\left\|K_{nj}\right\|\cdot\left\|\delta\right\|\leq|\delta|_{\max}. \quad (7)$$

Apply the inverse finite-integral transformation to (6), taking into account (7) and assuming that Δ integrates with the square, we obtain the error estimate:

$$\|\Delta\|\leq|\delta|_{\max}\left\|\sum_{n=1}^{\infty}K_{nj}\Big/\left|\sum_{k=0}^{l}a_{l-k}\sigma_{nj}^k\right|_{\min}\right\| \text{ or } \|\Delta\|\leq|\delta|_{\max}\left\|\sum_{n=1}^{\infty}\left|K_{nj}\right|_{\max}\Big/\left|\sum_{k=0}^{l}a_{l-k}\sigma_{nj}^k\right|_{\min}\right\|. \quad (8)$$

From the analysis (8), it can be seen that under the same conditions for the regulator that the error of the output vector is synthesized, the error of the influence of the approximation of the nonlinear part of the model and the number of eigenvalues and the choice of the type of kernels is formed. Studying the sensitivity separately to each of these factors is an important practical task for the final formation of an analytical selection criterion, but less important are their transparency and ease of calculation and unambiguous logical conclusions.

B. Model transformation. Until now, the direct problem of the dynamics of a nonlinear object for a generalized model described by a nonlinear differential equation has traditionally been posed and solved. However, in connection with the needs [7,8,10, 13] of quantitative grounds for the formation of logical conclusions, the task of finding a more informative result than the solution of a mathematical problem was set.

Let's pose the problem of a three-stage transformation of this model. Let's assume that its first stage is completed: the solution is formed as a recurrent sequence and its evaluation is given. As the second stage, we will present in an analytical form the dynamics of the difference between two successive approximations depending on the accuracy of the approximation of the nonlinearity of the object properties and the iteration number. Searching for the number of iterations that will ensure an error smaller than the given one is the third stage. Consider the recurrent schedule of problem (3) in which the linear operator $L_1(\Delta_p)$ of the form (6) is generalized. As a result of this transformation, for the error in *p*- that approximation we rewrite equation (3):

$$L_1(\Delta_p) = \delta(r,t) + \left.\frac{\partial L_2(\Phi)}{\partial \Delta}\right|_{\Phi_1} \Delta_p + \left.\frac{\partial^2 L_2(\Phi)}{\partial \Delta^2}\right|_{\Phi_1} \frac{\Delta_p \Delta_p}{2}\,. \tag{9}$$

Applying the finite-integral transformation (4) to both parts of (3) and (8) simultaneously using the properties of the Buniakovsky and Cauchy inequalities [22] and the kernel normalization conditions, we write:

$$\tilde{\Delta}_p \sum_{k=0}^{l} a_{l-k}\sigma_{nj}^k \le |\delta|_{\max} + \tilde{\Delta}_p \left| \left.\frac{\partial L_2(\Phi)}{\partial \Delta}\right|_{\Phi_1} \right|_{\max} + \tilde{\Delta}_{p-1} \left| \left.\frac{\partial^2 L_2(\Phi)}{\partial \Delta^2}\right|_{\Phi_1} \frac{\tilde{\Delta}_{p-1}}{2} \right|_{\max}. \tag{10}$$

The inverse transformation (9) gives:

$$\Delta_p \le \int_o^\infty K_{nj}(\sigma_j) \frac{|\delta|_{\max} + \tilde{\Delta}_{p-1} \left| \left.\frac{\partial^2 L_2(\Phi)}{\partial \Delta^2}\right|_{\Phi_1} \frac{\tilde{\Delta}_{p-1}}{2} \right|_{\max}}{\sum_{k=0}^{l} a_{l-k}\sigma_{nj}^k - \left| \left.\frac{\partial L_2(\Phi)}{\partial \Delta}\right|_{\Phi_1} \right|_{\max}} d\sigma_j\,. \tag{11}$$

Let's enter the notation:

$$A = \left\| \int_o^\infty \frac{K_{nj}(\sigma_j)|\delta|_{\max}}{\sum_{k=0}^{l} a_{l-k}\sigma_{nj}^k - \left| \left.\frac{\partial L_2(\Phi)}{\partial \Delta}\right|_{\Phi_1} \right|_{\max}} d\sigma_j \right\|; B = \frac{1}{2} \left\| \int_o^\infty K_{nj}(\sigma_j) \frac{\left| \left.\frac{\partial^2 L_2(\Phi)}{\partial \Delta^2}\right|_{\Phi_1} \right|_{\max}}{\sum_{k=0}^{l} a_{l-k}\sigma_{nj}^k - \left| \left.\frac{\partial L_2(\Phi)}{\partial \Delta}\right|_{\Phi_1} \right|_{\max}} d\sigma_j \right\|,$$

then, applying the norm to (11), we estimate two successive approximations:

$$\|\Delta_p\| \le \|A\| + |B|_{\max} \left|\tilde{\Delta}_{p-1}\right|_{\max} \|\Delta_{p-1}\|\,. \tag{12}$$

Their difference can be transformed by taking into account the equality the difference of norms for two functions to the norm of difference for these functions. Also, is assumed that the application of a finite-integral transformation and norm for functions of the image error in two iterations don't change the relation between them:

$$\tilde{\Delta}_p \left\| \Delta_{p-1} \right\| = \tilde{\Delta}_{p-1} \left\| \Delta_p \right\|.$$

Thus the difference between two norms of serial errors transformed by taking into account with described earlier pointed equality of the difference of norms and properties of transformation and norm for functions of the image error can be represented:

$$\left\| \Delta_{p+1} - \Delta_p \right\| \le \left[|B|_{\max} \left| \tilde{\Delta}_p - \tilde{\Delta}_{p-1} \right|_{\max} \right]^n \left\| \Delta_p + \Delta_{p-1} \right\|.$$

The sequential determination of such estimates by the inductive method leads to the representation of the differences for $n+1$ -th iteration and n through the first:

$$\left\| \Delta_{n+1} - \Delta_n \right\| \le \left[|B|_{\max} \left| \tilde{\Delta}_p - \tilde{\Delta}_{p-1} \right|_{\max} \right]^n \left\| \Delta_2 + \Delta_1 \right\|.$$

Let's introduce the relative error:

$$\varepsilon_{p+1} = \frac{\left\| \Delta_{p+1} - \Delta_p \right\|}{\left| \Delta_p \right|_{\max}}; \varepsilon_p = \frac{\left\| \Delta_p + \Delta_{p-1} \right\|}{2\left| \Delta_p \right|_{\max}},$$

then their dynamics from n iteration to the next $n+1$ will be presented:

$$\varepsilon_{n+1} \le 2\left[|B|_{\max} \left| \tilde{\Delta}_p - \tilde{\Delta}_{p-1} \right|_{\max} \right]^n \varepsilon_n$$

and their number before reaching an error value smaller than the specified error will be calculated as a whole part of the number:

$$n \ge \left[ln(\varepsilon_{n+1}) - ln(\varepsilon_n) - ln(2) \right] \big/ ln(|B|_{max}) + ln(\left| \tilde{\Delta}_p - \tilde{\Delta}_{p-1} \right|_{max}).$$

Thus, as a result of the procedure of justified transformations and actions, a method of transforming the shape of the model was formed, as a result of which we have three new formalized information quantities:

- first, the analytical expression of the solution approximation;
- secondly, the analytical expression of the upper limit of the iteration error;
- thirdly, an analytical expression of the number of iterations, which allows us to calculate it as the number, starting from which the error will be smaller than the specified one, which gives it the properties of a criterion AI for choosing a model.

Thus, such a calculation allows not only to present the solution as a recurrent sequence, but also to present information for analysis and automatically determine the number of iterations and the maximum error for each of them. Further, such a well-founded and proposed set of model transformation actions, presented in differential form to an algebraic sequence, will be called a model transformation method. The tool that determined the number of iterations at which the error becomes smaller than the specified one is called a boundary estimation tool. It is an AI tool for analyzing the quality of nonlinear models in differential form by its ability to make automatic conclusions. However, since it contains elements of information analysis and conclusion that are suitable to be carried out without human participation, it is an AI tool.

4.3 Assessment of the influence of the linearization error of the ACS object, the indicator vector on the model error

The model (3) with the generalized operator (2) and all the notations, assumptions and properties introduced in section 4.2 was considered for further determination of the influence of each of the factors - sources of error. Also, using the method of recurrent approximation [10, 13], we decompose the images formed by their action in the vicinity of the preimage Φ_1. Let us choose as Φ_1 the approximate solution of the boundary value problem that satisfies the linearized equation (3), which is obtained by approximation $L_2(\Phi)$ with a linear form $L_3(\Phi)$. As a result of this transformation, taking into account (8), after expansion into a series according to the method of recurrent approximation, in the neighborhood of the first approximation, we write down, taking into account the three component vector-indicator applied to the function and its first two derivatives:

$$L_1(\Delta_p) = V_1\delta(r,t) + V_2 \left.\frac{\partial L_2(\Phi)}{\partial \Delta}\right|_{\Phi_1} \Delta_p + V_3 \left.\frac{\partial^2 L_2(\Phi)}{\partial \Delta^2}\right|_{\Phi_1} \frac{\Delta_p \Delta_{p-1}}{2}, \tag{13}$$

where Δ_p and Δ_{p-1} is the solution error in the *p*-th and *p-1*-th approximation.

Applying the finite-integral transformation to (13), we write:

$$\tilde{\Delta}_p \sum_{k=0}^{l} a_{l-k}\sigma_{nj}^k = V_1 \int_a^b K_{nj}\delta dx_j + \int_a^b K_{nj}\Delta_p \left[V_2 \left.\frac{\partial L_2(\Phi)}{\partial \Delta}\right|_{\Phi_1} + V_3 \left.\frac{\partial^2 L_2(\Phi)}{\partial \Delta^2}\right|_{\Phi_1} \frac{\Delta_{p-1}}{2} \right] dx_j . \tag{14}$$

We apply the properties of Buniakovsky-Cauchy inequalities to equation (14) and give an estimate of the upper bound of the transformation error:

$$\tilde{\Delta}_p \le \frac{\left| V_1 \int_a^b K_{nj}\delta dx_j \right|_{\max} + \left\| \left[V_2 \left.\frac{\partial L_2(\Phi)}{\partial \Delta}\right|_{\Phi_1} + V_3 \left.\frac{\partial^2 L_2(\Phi)}{\partial \Delta^2}\right|_{\Phi_1} \frac{\Delta_{p-1}}{2} \right] \right\|_{\max} \int_a^b K_{nj}\Delta_p dx_j}{\sum_{k=0}^{l} a_{l-k}\sigma_{nj}^k} \tag{15}$$

or

$$\frac{\left| V_1 \int_a^b K_{nj}\delta dx_j \right|_{\min}}{\sum_{k=0}^{l} a_{l-k}\sigma_{nj}^k - \left\| \left[V_2 \left.\frac{\partial L_2(\Phi)}{\partial \Delta}\right|_{\Phi_1} + V_3 \left.\frac{\partial^2 L_2(\Phi)}{\partial \Delta^2}\right|_{\Phi_1} \frac{\Delta_{p-1}}{2} \right] \right\|_{\min}} \le \tilde{\Delta}_p \le \frac{\left| V_1 \int_a^b K_{nj}\delta dx_j \right|_{\max}}{\sum_{k=0}^{l} a_{l-k}\sigma_{nj}^k - \left\| \left[V_2 \left.\frac{\partial L_2(\Phi)}{\partial \Delta}\right|_{\Phi_1} + V_3 \left.\frac{\partial^2 L_2(\Phi)}{\partial \Delta^2}\right|_{\Phi_1} \frac{\Delta_{p-1}}{2} \right] \right\|_{\max}} \tag{16}$$

$$\Delta_p = \int_0^\infty \frac{K_{nj}}{\sum_{k=0}^{l} a_{l-k}\sigma_{nj}^k} \left\{ V_1 \int_a^b K_{nj}\delta dx_j + \int_a^b K_{nj}\Delta_p \left[\begin{array}{l} V_2 \left.\frac{\partial L_2(\Phi)}{\partial \Delta}\right|_{\Phi_1} + \\ + V_3 \left.\frac{\partial^2 L_2(\Phi)}{\partial \Delta^2}\right|_{\Phi_1} \frac{\Delta_{p-1}}{2} \end{array} \right] dx_j \right\} d\sigma_{nj} . \tag{17}$$

Expressions (15), (16) will allow us to estimate the upper and lower limits of the error value, and expression (17) allows us to trace its changes for different values of the state parameters. A detailed analysis of these expressions shows that their practical application can be carried out in two ways. According to the first of them, the gradual finding of estimates for each of the eigenvalues and the substitution of previous approximations, the transformed error values are found. After that, the inverse transformation of the error is found for their entire set. This expression is then used as a preliminary approximation, and the algorithm is repeated.

According to the second approach, immediately after the entire set of eigenvalues, according to expression (16), the *p*-th approximation of the error is found, with a zero value of the previous *p-1* -th approximation. Further, after performing the integration operation, this result is used again, after substitution to expression (16). This obtained result is taken to find the next approximation of the inverse transformation. And further, according to this algorithm, the process is repeated until the convergence of the sequence with the specified accuracy is achieved. It should also be noted that the introduction of the vector-indicator links the qualitative and linguistic variables, with the help of which the production-managing rules are formed, according to which the decision-maker operates, with the quantitative variables. The latter ensures the connection of the modules of the hybrid SPPR with the modules built according to the principles of neural networks, including recurrent ones.

4.4 Formation of the kinematics model of the manipulator based on the analytical solution of the inverse problem.

To perform the functions of mobile robotics [4], manipulators are installed on their board. The kinematic diagram of the articulated manipulator is presented in Fig. 1.

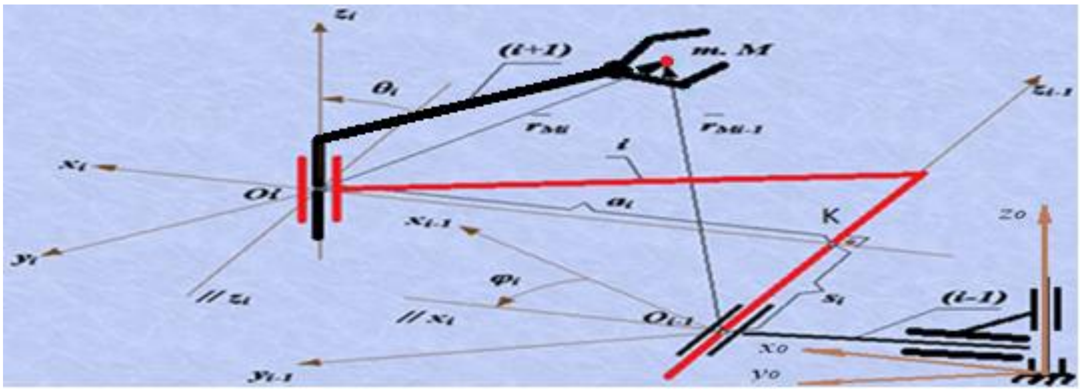

Figure 1: The kinematic diagram of a manipulator with five degrees of mobility.

Development of the sensory systems architecture and models for describing the kinematics and dynamics and determining suitability to the application of the analytical solution of the inverse kinematics problem is a main problem of task with multiple solutions. In the search for a stable and unique solution to the inverse kinematics problem, an approach known as the inverse matrix method was considered [23]. The analysis of the matrix equation shows that in order to ensure the equality of the matrices, the condition of equality of nine elements with the same name must be fulfilled, that is, a system of nine equations:

$$\begin{cases} C\varphi n_x + S\varphi n_y = C\psi; \\ C\varphi s_x + S\varphi s_y = -S\psi; \\ C\varphi a_x + S\varphi a_y = 0; \\ -S\varphi n_x + C\varphi n_y = C\theta S\psi; \\ -S\varphi s_x + C\varphi s_y = C\theta C\psi; \\ -S\varphi a_x + C\varphi a_y = -S\theta; \\ n_z = S\theta S\psi; \\ s_z = S\theta C\psi; \\ a_z = C\theta. \end{cases} \quad (18)$$

The resulting structure and composition of system equations and the search for its solution and their errors were analyzed. The system contains either three unknown vectors $\bar{a}, \bar{s}, \bar{n}$, each of which has three components as projections on the direction of three axes, as well as three angles φ, θ, ψ. In this regard, it should be supplemented with three more independent equations. Otherwise, it is regrouped into three equations with three unknowns and at least one of which is heterogeneous. This algorithm was obtained based on the idea of reducing a system of nine equations with three unknowns to a system of three heterogeneous equations with three unknowns. For the analysis, in contrast to the existing approaches to the solution of the inverse kinematics problem, we write down the condition for the connection of the components of the vector ***a*** and the approach angle:

$$C\varphi = -S\varphi \frac{a_y}{a_x}. \tag{19}$$

This condition leads the first two equations to a structurally similar form:

$$\left\{ S\varphi\left(n_y - \frac{a_y n_x}{a_x}\right) = C\psi; \qquad S\varphi\left(s_y - \frac{a_y s_x}{a_x}\right) = -S\psi \right. .$$

The latter, according to the theorem on the sum of the squares sine and cosine of one angle, will become:

$$S^2\varphi\left[\left(n_y - \frac{a_y n_x}{a_x}\right)^2 + \left(s_y - \frac{a_y s_x}{a_x}\right)^2\right] = C^2\psi + S^2\psi.$$

Its further simplification, as the idea of reducing variables, excludes from the equation the trigonometric function of the trim angle ψ, which is an unknown quantity. As a result, the sine of the grip approach angle is immediately found:

$$S\varphi = \left[\left(n_y - \frac{a_y n_x}{a_x}\right)^2 + \left(s_y - \frac{a_y s_x}{a_x}\right)^2\right]^{-1/2}. \tag{20}$$

Substitution (19), defined by the third equation of the original system (18), after multiplying by the sine of the angle θ of the relationship between the sine and cosine of the angle φ, defined by the third equation and after multiplying by the sine of the angle and simple algebraic transformations, taking into account the equality of the left parts, we write:

$$\frac{C^2\theta - 1}{a_x + a_y \dfrac{a_y}{a_x}} = C\theta s_z \frac{1}{s_x + s_y \dfrac{a_y}{a_x}},$$

which reduces to a quadratic equation:

$$C^2\theta - C\theta \frac{s_z(a_x^2 + a_y^2)}{(s_x a_x + s_y a_y)} - 1 = 0.$$

The latter is not rational. Its special drawback is the duality of the root. The decision to discard the minus sign is not unequivocally obvious for the general case. Substitution of the projection of the vector a on the Z axis simplifies the quadratic equation, which allows you to determine the cosine of the angle θ through the known orientation vectors:

$$C\theta = \frac{(s_x a_x + s_y a_y)}{a_z(s_x a_x + s_y a_y) - s_z(a_x^2 + a_y^2)}. \tag{21}$$

Now from the fifth equation, taking into account equations three and nine, after simple algebraic transformations, we write:

$$S\psi = -\frac{S\varphi(n_x a_x + n_y a_y)}{a_x C\theta} = -\frac{S\varphi(n_x a_x + n_y a_y)}{a_x a_z}. \tag{22}$$

Further examination of the equations of the system shows that the solution used all nine homogeneous equations of the initial system (18), which was supplemented by two non-homogeneous equations of the sum of the squares of the sine and cosine of the angles ψ and θ. Trigonometric functions from the value of trim ψ and roll θ angles are calculated even at small course angles φ, since the operation of dividing by small values is excluded. Thus, three spatial angles are calculated based on the values of the projections of three mutually orthogonal vectors of the grasping position:

$$\begin{cases} \varphi = \arcsin\left[\left(n_y - \frac{a_y n_x}{a_x}\right)^2 + \left(s_y - \frac{a_y s_x}{a_x}\right)^2\right]^{-1/2}; \\ \psi = -\arcsin\frac{S\varphi(n_x a_x + n_y a_y)}{a_x a_z}; \\ \theta = \arccos\frac{(s_x a_x + s_y a_y)}{a_z(s_x a_x + s_y a_y) - s_z(a_x^2 + a_y^2)}. \end{cases} \tag{23}$$

Thus, the application of two heterogeneous equations ensured the unity of the solution of the inverse kinematics problem.

5. Modeling and discussion of results

A. Modeling the influence of the number of eigenvalues and model parameters. For clarity and ease of presentation of the results of the modeling process, consider a model for which the linear operator is simplified. Assume that the transformation $L_1(\Phi)$ is a linear function from the function Φ transformed with the same kernel K_{nj} and eigenvalues σ_{nj}. The analysis of rough (15), (16) and more accurate estimates (17) shows that if the linearization error is a finite value , then the estimate of the decision error is also a finite value. The most interesting fact is that if the series of eigenvalues is bounded only from below, then the error estimates are sharply reduced, and the upper limit only simplifies the calculation of the sums of the series, as was observed, for example, for the sine and cosine transformations. Thus, analyzing the expression (17), it is possible to conclude about the possibility of applying the methods of finite-integral transformations to nonlinear problems, if there are $\sigma_n \geq \sqrt{|\delta|_{\max} / |\Phi|_{\max}}$. A special selection of kernels only with values of eigenvalues satisfying this inequality allows obtaining an approximate solution with a relative error much less than unity. It is convenient to choose the linearization coefficients for the limited search functions both above and below by entering the dimensionless search function Φ/Φ_{max}, based on the criterion of equality of both the deviation itself at one of the points or in some set of them, and the deviation of the integrals from its dimensionless value on the interval [0,1]. However, the analysis of expressions (5) shows that if the integral of the square of the discrepancy is linearized, the error is significantly reduced.

Thus, an automatic inference tool is substantiated, which will allow obtaining an estimate of the error of the solutions, which also allows the use of norm estimation expressions as criteria. So, for example, for a hybrid decision-making support system, a model database management system, an algorithm database management system, a system of algorithms based on control rules, including by the value of the maximum error estimate, will allow the selection of a type of model or algorithm based on the value of the specified accuracy and control their application according to by the values of the components of the vector-indicator. To study the influence of the number of eigenvalues l on the error norm $\|\Delta\|$ and relative error ε, consider the second-order model for the set of coefficient ratio values a_1/a_0. The simulation results are presented in table 1.

Table 1

Analysis of the influence of the number of eigenvalues on the estimation of the value of the norm of error

	$a_1/a_0=0{,}1$		$a_1/a_0=0{,}5$		$a_1/a_0=1$	
l	$\|\Delta\|$	ε	$\|\Delta\|$	ε	$\|\Delta\|$	ε
1	1,74223	-7,80596	1,66672	-8,7778	1,58227	-9,6009
2	0,37475	0,577101	0,36478	0,86012	0,35324	1,20745
3	0,17351	0,5699	0,16899	1,05388	0,16379	1,61623
4	0,09827	0,446845	0,09606	0,86653	0,09350	1,35691
5	0,06309	0,333402	0,06187	0,66467	0,06043	1,05534
6	0,04390	0,252098	0,04315	0,51124	0,04227	0,81929
7	0,03229	0,195167	0,03180	0,40028	0,03122	0,64568
8	0,02474	0,154672	0,02441	0,31973	0,02401	0,51822
9	0,01956	0,125173	0,01932	0,26024	0,01904	0,42334
10	0,01585	0,103161	0,01568	0,21540	0,01546	0,3514

As evidenced by the analysis of simulation results, the effect of increasing the number of eigenvalues from one to ten reduces the maximum possible error from tens of percent to units. This influence is more significant compared to the influence of coefficients. A ten-fold change in the order of the ratio leads to an increase in the error of only less than four times. The latter shows that the factor of the number of eigenvalues for forming the kernels of integral transformations is more important and it primarily determines the amount of error when modeling processes. The analyticity of expressions for estimating the error of the model's predicted behavior makes them suitable for simple, quick calculations. The latter gives such assessments an advantage for selecting them as criteria for the SPD of the hybrid architecture of underwater technologies. Interactive modeling of dynamics for the received forms (11)-(12) is presented in Fig. 2.

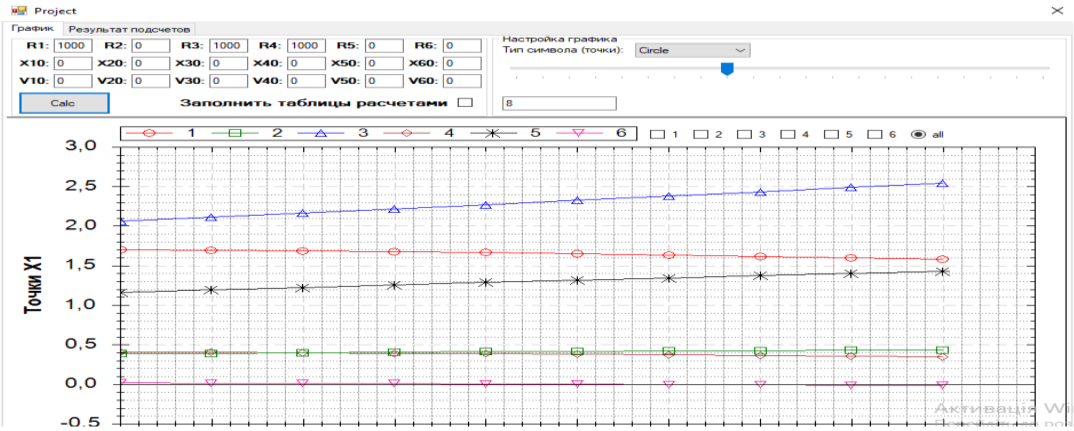

Figure 2: Screen image of robot motion simulation results

The program allows to set forces, moments of forces, initial positions, velocities and accelerations as three-component vectors, output data in the form of graphs and tables upon request. In addition, a tool (tool box) is used that outputs data in an interval as a function starting from a specific moment in time from a given interval.

B Modeling of angular positions and solutions of the inverse kinematics problem

The solutions of the inverse kinematics problem, which do not contain the division into small values of the required angles and are suitable for express calculations in the form (18), are not identical in form. In this regard, for comparison, we will conduct simulations and determine their correspondence. Sets of orientation vectors n, s, a were chosen for modeling, overlapping the ranges of actual changes inherent in inverse kinematics problems. The list of data used for calculations is given in table 2

Table 2

A set of values of the components of the vectors used for simulation

№	n_x	n_y	n_z	s_x	s_y	s_z	a_x	a_y	a_z
1	1	0	0	0	-1	8,27E-5	0	-8,3E-05	-1
2	1	1,07E-5	0,00061	-1,1E-5	-0,9994	0,03498	0,00061	-0,0349	-0,9994
3	0,99999	8,51E-5	0,00244	-8,5E-5	-0,9976	0,06979	0,00244	-0,0698	-0,9976
4	0,99999	0,00029	0,00547	-0,0003	-0,9945	0,10447	0,00547	-0,1045	-0,9945
5	0,99995	0,00068	0,00971	-0,0007	-0,9903	0,13891	0,00971	-0,1389	-0,9903
6	0,99989	0,00132	0,01513	-0,0013	-0,9849	0,17307	0,01513	-0,1731	-0,9848
7	0,99976	0,00227	0,02173	-0,0023	-0,9784	0,20685	0,02173	-0,2069	-0,9781
8	0,99956	0,00359	0,02948	-0,0036	-0,9707	0,24019	0,02948	-0,2402	-0,9703
9	0,99925	0,00534	0,03835	-0,0053	-0,9629	0,27304	0,03835	-0,2730	-0,9612
10	0,99880	0,00756	0,04832	-0,0076	-0,9522	0,30529	0,04833	-0,3053	-0,9510
11	0,99818	0,01031	0,05937	-0,0103	-0,9415	0,3369	0,05937	-0,3369	-0,9397

A set of values of the vectors ***n, s, a*** is taken for modeling and checking the correctness and uniformity of the solutions. Values of angles Θ, φ, ψ, that is the result of a set of turns, taken as benchmarks, and their values, which are obtained as a solution to (23), are compared to the benchmarks. The result of the comparison in relative values is presented in the table. 3 (analysis of the relative error of the solutions of the inverse dynamics problem). In the table 3 subscripts p at the value of each of the angles indicate values determined by analytical solutions of expressions (23). The relative error for each of the angles is marked with a corresponding subscript indicating the angle are shown in in table 3.

Table 3

Absolute and relative errors for gripper position angles

№	Θ	θ_p	ε_θ	φ	φ_p	ε_φ	ψ	ψ_p	ε_ψ
1	3,1415	3,1415	-3,55E-13	0	#ДЕЛ/0!	#ДЕЛ/0!	0	#ДЕЛ/0!	#ДЕЛ/0!
2	3,1066	3,1066	-3E-16	0,0174	0,0174	0	0,0174	0,0174	0
3	3,0717	3,0717	-9E-16	0,0349	0,0349	-2E-16	0,0349	0,0349	0
4	3,0368	3,0368	7E-16	0,0523	0,0523	4E-16	0,0523	0,0523	5E-16
5	3,0019	3,0019	3E-16	0,0698	0,0698	2E-16	0,0698	0,0698	2E-16
6	2,967	2,967	0	0,0872	0,0872	0	0,0872	0,0872	0
7	2,9321	2,9321	-2E-16	0,1047	0,1047	-1E-16	0,1047	0,1047	-1E-16
8	2,8972	2,8972	0	0,1221	0,1221	2E-16	0,1221	0,1221	1E-16
9	2,8623	2,8623	0	0,1396	0,1396	-2E-16	0,1396	0,1396	0
10	2,8274	2,8274	-2E-16	0,157	0,157	0	0,157	0,157	4E-16
11	2,7925	2,7925	-2E-16	0,1744	0,1744	5E-16	0,1744	0,1744	6E-16

The following original statements about rotations of vectors ***n, s, a*** were used for modeling. Rotation of the vector ***n*** around the OY axis by an angle $\Delta\varphi$. The vector ***a*** will also return to the angle $\Delta\varphi$. Rotation of the vector ***n*** around the OZ axis by an angle $\Delta\psi$. The vector s will also turn by the angle $\Delta\psi$. Rotation of the vector ***n***, as well as rigidly connected vectors ***s*** and ***a*** around the OX axis by an angle $\Delta\Theta$. The vector ***s*** and ***a*** will turn to the angle $\Delta\Theta$.

As the data analysis (Table 3) shows, the relative error is 13–16 of the order of magnitude. The latter only confirms the assumption that the reason for the non-unity was an attempt to solve a system of nine equations with three unknown by direct methods. Correct reduction of the system to the canonical form: to three equations with three unknowns, gives a single solution. The obtained solutions are analytical and allow their use for building analytical models of the synthesis of control influences for a multi-link manipulator. The expressions of solutions for angles themselves are simple in form and suitable for express calculations. The further application of such solutions to solve problems of the dynamics of manipulators or other elements of robotic systems opens up new possibilities. The building of analytical model dynamics allows research and design using effective analytical operational research methods. The analyticity of the models built based on the analytical solution of the inverse kinematics problem will simplify external wireless control sensory systems. The future development of applications based on Android smartphones will increase the necessity of analytical single solutions to inverse problems, including kinematics problems.

Conclusions

1. The platform of mobile robots for experimental research means of intellectualization and modification of sensory control systems can be used if it is built based on an autonomous navigation system. For example, if it is built on the JetRacer platform using the NVIDIA Jetson Nano.
2. Analytical expressions for estimating the speed of converges of error in the description of a nonlinear object are constructed using the methods of finite-integral transformations for continuous models. Estimates of the norm of the error of the solution of the modeling problem for an infinite and finite number of eigenvalues do not depend on the type of the transformation kernel but are determined to a greater extent by the number of eigenvalues of the problem and the error of approximation of nonlinear terms and the properties of the object for an arbitrary operator and an arbitrary transformation kernel. The learning and determination of values in three-level comparators and vector-indicators based on these results play key`s role in application of them as a tool of AI. This approach expanded informativity of solutions and obtained:

- the analytical expression of the solution;
- the analytical expression of the upper limit of the iteration error;
- an analytical expression of the number of iterations, which allows us to calculate it as the number, starting from which the error will be smaller than the required one, and which opens the properties as of a criterion AI for choosing a model.

3. The kinematic model of the manipulator based on the analytical solution of the inverse kinematics problem for unity solution simultaneously with a sensory system of mobile robots opens the possibility of correction and calibration of sensors according due to the received unity solution. A solution was obtained that meets the reference conditions, does not contain the division into small values of the sines of the sought angles and is suitable for analytical models and express calculations, especially for robotic systems of external wireless control through software applications based on Android smartphones.

Reference

[1]. Tesla Autopilot: What is it and how does it work? Here's everything you may want to know Read more at: https://economictimes.indiatimes.com/news/international/us/tesla-autopilot-what-is-it-and-how-does-it-work-heres-everything-you-may-want-to-know/articleshow/101601035.cms?from=mdr . Electronic resors (Date of Application: 25.02.2024).

[2]. Under the Hood of Uber ATG's Machine Learning Infrastructure and Versioning Control Platform for Self-Driving Vehicles. https://www.uber.com/en-UA/blog/machine-learning-model-life-cycle-version-control/ March 4, 2020 / Global/ (Date of Application : 28.02.2024).

[3]. A. Pequeño-Zurro *et al*., "Proactive Control for Online Individual User Adaptation in a Welfare Robot Guidance Scenario: Toward Supporting Elderly People," in *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 53, no. 6, pp. 3364-3376, June 2023, doi: 10.1109/TSMC.2022.3224366. keywords: {Robots;Older adults;Legged locomotion;Behavioral sciences;Navigation;Visualization;Adaptation models;Adaptive behavior;guide robot;human–robot interaction;neural control;service mobile robot;social robot;welfare robot},

[4]. A. Trunov, P. Kazan, V. Alieksieiev, O. Korolova, O. Sliusarenko, I. Dronyuk, Functioning Model of The Ground Robotic Complex, in: Proceedings of The International Scientific and Technical Conference on Computer Sciences and Information Technologies (CSIT'21), IEEE, 2, p. 128–131. 2021, doi: 10.1109/CSIT52700.2021.9648595.

[5]. Trounov A.N. Application of sensory modules for adaptive robot. *Proc. 3rd Int. Conf. on Robot Vision and Sensory Control*, 9-11, Oct. London 1984. Pp. 284-294.

[6]. How HC-SR04 Ultrasonic Sensor Works & Interface It With Arduino. URL: https://lastminuteengineers.com/arduino-sr04-ultrasonic-sensor-tutorial/ (дата звернення: 14.12.2023).

[7]. Kargin, A., & Ivaniuk, O. Autonomous robot motion control situational planning model. *Advanced Information Systems*. 2020. Vol. 4. No. 3. P. 41–51. DOI:10.20998/2522-9052.2020.3.05.

[8]. Ivaniuk O. Navigation of Autonomous Systems based on Situation Control with Dynamic Replanning. *Information Processing Systems*. 2020. № 3 (162). P. 44–51. doi: 10.30748/soi.2020.162.05.

[9]. Oliveira, M., Castro, A., Madeira, T., Pedrosa, E., Dias, P., & Santos, V. A ROS framework for the extrinsic calibration of intelligent vehicles: A multi-sensor, multi-modal approach. Robotics and Autonomous Systems. 2020. DOI: 10.1016/j.robot.2020.103558.

[10]. Trunov A. Recurrent Approximation as the Tool for Expansion of Functions and modes of operation of Neural Network. Vol. 5 No. 4 (83) (2016).pp.41-48: Mathematics and Cybernetics - applied aspects. DOI: https://doi.org/10.15587/172940 61.2016.81298

[11]. Krainyk Y. Design and Implementation of Image Sensor Data Capture Based on FPGA December 2023 SN Computer Science 5(1) 95 DOI: 10.1007/s42979-023-02433-5

[12]. Khnissi, K., Ben Jabeur, C., & Seddik, H. A smart mobile robot commands predictor using recursive neural network. *Robotics and Autonomous Systems*. 2020. Vol. *131*. DOI: 10.1016/j.robot.2020.103593.

[13]. Trunov, Alexander. "Recurrent Transformation of the Dynamics Model for Autonomous Underwater Vehicle in the Inertial Coordinate System." *Eastern-European Journal of Enterprise Technologies*, vol. 2, no. 4, 2017, pp. 39-47, doi:10.15587/1729-4061.2017.95783

[14]. Kondratenko, Y.P. Robotics, Automation and Information Systems: Future Perspectives and Correlation with Culture, Sport and Life Science. In: ***Decision Making and Knowledge Decision Support Systems***, Lecture Notes in Economics and Mathematical Systems, vol. 675, A. M. Gil-Lafuente, C. Zopounidis, Eds. Springer International Publishing Switzerland, pp. 43–56 (2015).

[15]. Kondratenko Y., Atamanyuk I., Sidenko I., Kondratenko G., Sichevskyi S. Machine learning techniques for increasing efficiency of the robot's sensor and control information processing. *Sensors*. 2022. 22(3). 1062. 31 p. https://doi.org/10.3390/s22031062

[16]. Solesvik M., Kondratenko Y., Kondratenko G., Sidenko I., Kharchenko V., Boyarchuk A.: Fuzzy decision support systems in marine practice. In: Fuzzy Systems 2017. IEEE Int. Conf. DOI: 10.1109/FUZZ-IEEE.2017.8015471

[17]. Kupin A., Kuznetsov D., Muzuka, Paraniuk D., Serdiuk O., Suvoruv DvornikovV. The concept of a modular cyberphysical systemfor the early diagnosis of energy equipment. Eastern-European Journal of Enterprise Technologies, 4/2 (94) 2018. P. 72-79. DOI:10.15587/1729-4061.2018.139644

[18]. M. Dyvak, A. Pukas and O. Kozak, "Tolerance estimation of parameters set of models created on experimental data," 2008 International Conference on "Modern Problems of Radio Engineering, Telecommunications and Computer Science" (TCSET), Lviv, UKraine, 2008, pp. 24-26.

[19]. M. Dyvak, Algorithms of parallel calculations in task of tolerance ellipsoidal estimation of interval model parameters [Electronicresource] / M. Dyvak, P. Stakhiv, A. Pukas // Bulletin of the PolishAcademy of Sciences: Technical Sciences. – 2012. – № 1. – P. 159-164. DOI: 10.2478/v10175-012-0022-9

[20]. T. Toosi, M. Sirola, J. Laukkanen, , M. van Heeswijk, J. Karhunen, (2019). Method for detecting aging related failures of process sensors via noise signal measurement. International Journal of Computing, 18(2), 135-146. https://doi.org/10.47839/ijc.18.2.1412

[21]. Herman Fesenko, Vyacheslav Kharchenko, Anatoliy Sachenko, Robert Hiromoto and Volodymyr Kochan. An Internet of Drone-based Multi-Version Post-severe Accident Monitoring System: Structures and Reliability. In book Dependable IoT for Human and Industry - Modeling, Architecting, Implementation. Editors: Vyacheslav Kharchenko, Ah Lian Kor and Anrzej Rucinski. River Publishers, 2018, Pp.197-217. https://doi.org/10.1201/9781003337843-12

[22]. R. Bellman, "On the Approximation of Curves by Line Segments Using Dynamic Programming," Communications of the ACM, Vol. 4, No. 6, 1961, p. 284.

[23]. Цвіркун Л.І., Грулер Г. Робототехніка та мехатроніка: навч. посіб. Під заг. ред. Л.І. Цвіркуна; 3-є вид., переробл. і доповн. Дніпро : НГУ, 2017. 224 с.